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MATDIP401

Fourth Semester B.E. Degree Examination, June 2012

Advanced Mathematics - II

Time: 3 hrs. Max. Marks:100

Note: *Answer any FIVE full questions.*

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

1 a. Find the angles between any two diagonals of a cube. **(06 Marks)**

b. Find the equations of two planes, which bisect the angles between the planes $3x - 4y + 5z = 3$, $5x + 3y - 4z = 9$. **(07 Marks)**

c. Find the image of the point (1, 2, 3) in the line $\frac{x+1}{2} = \frac{y-3}{3} = -z$ **(07 Marks)**

2 a. Find the equation of the plane through the point (1, -1, 0) and perpendicular to the line $2x + 3y + 5z - 1 = 0 = 3x + y - z + 2$. **(06 Marks)**

b. Find the value of k such that the line $\frac{x}{k} = \frac{y-2}{2} = \frac{z+3}{3}$ and $\frac{x-2}{2} = \frac{y-6}{3} = \frac{z-3}{4}$ are coplanar. For this k find their point of intersection. **(07 Marks)**

c. Find the distance of the point (1, -2, 3) from the plane $x - y + z = 5$ measured parallel to the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$. **(07 Marks)**

3 a. Show that the position vectors of the vertices of a triangle $\vec{a} = 3(\sqrt{3}\hat{i} - \hat{j})$, $\vec{b} = 6\hat{j}$, $\vec{c} = 3(\sqrt{3}\hat{i} + \hat{j})$ form an isosceles triangle. **(06 Marks)**

b. Find the unit normal to both the vectors $4\hat{i} - \hat{j} + 3\hat{k}$ and $-2\hat{i} + \hat{j} - 2\hat{k}$. Find also the sine of the angle between them. **(07 Marks)**

c. Prove that the position vectors of the points A, B, C and D represented by the vectors $-\hat{j} - \hat{k}$, $4\hat{i} + 5\hat{j} + \hat{k}$, $3\hat{i} + 9\hat{j} + 4\hat{k}$ and $-4\hat{i} + 4\hat{j} + 4\hat{k}$, respectively are coplanar. **(07 Marks)**

4 a. Find the value of λ so that the points A(-1, 4, -3), B(3, 2, -5), C(-3, 8, -5) and D(-3, λ, 1) may lie on one plane. **(06 Marks)**

b. If $\vec{a}, \vec{b}, \vec{c}$ are the position vectors of points A, B, C, prove that $(\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a})$ is a vector perpendicular to the plane of triangle ABC. **(07 Marks)**

c. Find a set of vectors reciprocal to the set $2\hat{i} + 3\hat{j} - \hat{k}$, $\hat{i} - \hat{j} - 2\hat{k}$, $\hat{i} + 2\hat{j} + 2\hat{k}$. **(07 Marks)**

5 a. Find the maximum directional derivative of $\log(x^2 + y^2 + z^2)$ at (1, 1, 1). **(06 Marks)**

b. Find the unit normal vector to the curve $\vec{r} = 4\sin t\hat{i} + 4\cos t\hat{j} + 3t\hat{k}$. **(07 Marks)**

c. Show that $\vec{F} = \frac{x\hat{i} + y\hat{j}}{x^2 + y^2}$ is both solenoidal and irrotational. **(07 Marks)**

6 a. Find the Laplace transforms of $\sin^2 3t$ and $\sqrt{t}$. **(06 Marks)**

b. Find L[f(t)], given that $f(t) = \begin{cases} t-1 & 0 < t < 2 \\ 3-t & t > 2 \end{cases}$. **(07 Marks)**

c. Find the Laplace transform of $e^{2t}\cos t + t\,e^{-t}\sin 2t$. **(07 Marks)**

7 a. Find the Laplace transform of $\int_0^t \cos 2(t-u)\cos 3u\,du$. **(06 Marks)**

b. Find the inverse Laplace transform of

i) $\dfrac{s+1}{s^2-s+1}$ ii) $\dfrac{1}{s(s^2+a^2)}$. **(14 Marks)**

8 a. Find the inverse Laplace transform by using convolution theorem of $\dfrac{1}{(s^2+a^2)^2}$. **(10 Marks)**

b. By applying Laplace transform, solve the differential equation $\dfrac{d^2y}{dt^2}+5\dfrac{dy}{dt}+6y=5e^{2t}$. Subject to the conditions y(0) = 2 , y'(0) = 1. **(10 Marks)**

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